Dividing (6) by (5),

$$\tan\varphi=\frac{\mu a^2}{a^2+9(c-a)^2}\ldots.(7).$$

Also solved by *G. R. DEAN*, and *G. B. M. ZERR*.

114. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Prove that the *inclination* of a perfectly rough inclined plane must be $\theta=\sin^{-1}[e^2/(2-e^2)]$, in order that an ellipse of minimum eccentricity e may be capable of resting in equilibrium on the plane.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The center of the ellipse will be vertically over the point of support; also the vertical through the point of support and the parallel to the plane through the center will form conjugate diameters.

$\therefore$ The acute angle between these diameters$=\frac{1}{2}\pi-\theta$. Since the potential energy is greatest at a point bordering on motion, the major axis will bisect the angle $\frac{1}{2}\pi-\theta$; hence the major axis makes with the conjugate diameters angles $\frac{1}{4}\pi-\frac{1}{2}\theta$ and $\frac{1}{4}(3\pi)+\frac{1}{2}\theta$.

We have for conjugate diameters, then, this condition,

$$a^2\sin^2[\tfrac{1}{4}\pi-\tfrac{1}{2}\theta]\sin[\tfrac{1}{4}(3\pi)+\tfrac{1}{2}\theta]+b^2\cos[\tfrac{1}{4}\pi-\tfrac{1}{2}\theta]\cos[\tfrac{1}{4}(3\pi)+\tfrac{1}{2}\theta]=0,$$

$$\text{or } a^2\sin^2(\tfrac{1}{4}\pi-\tfrac{1}{2}\theta)-b^2\cos^2(\tfrac{1}{4}\pi-\tfrac{1}{2}\theta)=0.$$

$$\therefore \frac{a^2-b^2}{a^2}=e^2=\frac{2\sin\theta}{1+\sin\theta} \text{ or } \sin\theta=\frac{e^2}{2-e^2}.$$

$$\therefore \theta=\sin^{-1}[e^2/(2-e^2)].$$

115. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A vessel in the shape of a parallelopiped, filled with water, has in its horizontal bottom a rectangular opening, whose dimensions are a and b, which is shut up by a slider. Supposing this slider to be opened with a uniform motion in the direction of a. To find the depth of the water in the vessel after the time T at the moment when the slider has passed through the space a, a denoting the horizontal section of the water in the vessel.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let h=height of parallelopiped, K=area of bottom, x=depth of water at any time, y=the distance the slider has opened at any time.

Then average area of orifice$=b\int_0^a ydy/\int_0^a dy=\frac{1}{2}ab$.

$$\therefore t=-\frac{2K}{ab\sqrt{2g}}\int\frac{dx}{\sqrt{x}}=-\frac{4K\sqrt{x}}{ab\sqrt{2g}}+C.$$

Since $x=h$ when $t=0$, $C=\dfrac{4K\sqrt{h}}{ab\sqrt{2g}}$.

$$\therefore\ t=T=\frac{4K}{ab\sqrt{2g}}(\sqrt{h}-\sqrt{x}).\quad \therefore\ x=\left(\frac{4K\sqrt{h}-Tab\sqrt{2g}}{4K}\right)^2.$$

AVERAGE AND PROBABILITY.

---

98. Proposed by REV. PREBENDARY WHITWORTH, A. M.

A has £m and B has £n. They play for points until one of them has lost all his money. If α and β be the respective chances that A and B win any point, the expectation of the number of points played will be

$$\frac{n\alpha^n(\alpha^m-\beta^m)-m\beta^m(\alpha^n+\beta^n)}{(\alpha-\beta)(\alpha^{m+n}-\beta^{m+n})}.$$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let A_m=A's chance of winning, B_n=B's chance of winning.

Then nA_m, mB_n=A's and B's expectation, respectively.

$\therefore$ Expectation of number of points played$=E$.

Then $E=(nA_m-mB_n)/(\alpha-\beta)$.

Let A_x=A's chance when he has x pounds and B has $m+n-x$ pounds.

$$\therefore\ A_x=\frac{\beta}{\alpha+\beta}A_{x-1}+\frac{\alpha}{\alpha+\beta}A_{x+1}.$$

$A_x-A_{x-1}=(\alpha/\beta)(A_{x+1}-A_x)$. Giving x successive values from 1 to x we get $A_1-A_0=(\alpha/\beta)(A_2-A_1)$, $A_2-A_1=(\alpha/\beta)(A_3-A_2)$, etc.

By continued multiplication we get $A_1-A_0=(\alpha/\beta)^{x-1}(A_x-A_{x-1})$ or $A_x-A_{x-1}=(\beta/\alpha)^{x-1}(A_1-A_0)$.

Give x successive values from 1 to x and add

$$A_x-A_0=(A_1-A_0)[1+\beta/\alpha+(\beta/\alpha)^2+\ldots\ldots+(\beta/\alpha)^{x-1}].$$

But $A_0=0$. $\therefore\ A_x=A_1[1-(\beta/\alpha)^x]/[1-(\beta/\alpha)]$.

$A_{m+n}=1$. $\therefore\ 1=A_1[\alpha^{m+n}-\beta^{m+n}]/[\alpha^{m+n-1}(\alpha-\beta)]$.

$\therefore\ A_1=[\alpha^{m+n-1}(\alpha-\beta)]/(\alpha^{m+n}-\beta^{m+n})$.

$\therefore\ A_x=[\alpha^{m+n-1}(\alpha^x-\beta^x)]/[\alpha^{x-1}(\alpha^{m+n}-\beta^{m+n})]$.

$\therefore\ A_m=[\alpha^n(\alpha^m-\beta^m)]/(\alpha^{m+n}-\beta^{m+n})$.

Similarly, $B_n=[\beta^m(\alpha^n-\beta^n)]/(\alpha^{m+n}-\beta^{m+n})$.

$$\therefore\ E=\frac{n\alpha^n(\alpha^m-\beta^m)-m\beta^m(\alpha^n-\beta^n)}{(\alpha-\beta)(\alpha^{m+n}-\beta^{m+n})}.$$

99. Proposed by E. B. SEITZ.

A point is taken at random in the surface of a given circle, and from it a line equal in length to the radius is drawn, so as to lie wholly in the surface of the circle. Find the chance that the line intersects in a given diameter. [No. 135, *The Analyst.*]